ON A GENERALIZATION OF A THEOREM OF DINI ON SEQUENCES OF CONTINUOUS FUNCTIONS.

BY DR. T. H. HILDEBRANDT.

(Read before the American Mathematical Society at Chicago, April 10, 1914.)

WE propose in this note to give a generalization of the following theorem of Dini*: "If a monotonic sequence of functions continuous on a closed interval converges to a continuous function, the convergence is uniform."

The double sequence analogue of this theorem proves to be of importance in our generalization. We embody it in the following

LEMMA. *If a double sequence a_{mn} is monotonic non-decreasing in m for every n, and if $L_m L_n a_{mn} = L_n L_m a_{mn}$, all the limits being supposed to exist, then $L_m a_{mn}$ and $L_n a_{mn}$ converge uniformly and the double limit $L_{mn} a_{mn}$ exists and is equal to the iterated limits.*

The proof of the uniformity of convergence of $L_m a_{mn}$ is part of Theorem I of the paper by the author in the *Annals of Mathematics*, series 2, volume 14, page 81. This uniformity has as a direct consequence the existence of the double limit equal to the iterated limits, which in turn implies the uniformity of $L_n a_{mn}$.

For the purposes of generalization, consider a class $\mathfrak{O}$ of elements unconditioned excepting for the existence within the class of some definition of limit, i. e., some means of determining whether a sequence of elements has a limit and what this limit is.† Then it is possible to define the concepts *limiting element, closed and compact* relative to subclasses $\mathfrak{R}$ of $\mathfrak{O}$.‡ Also if μ is a real-valued function on $\mathfrak{R}$, we can define the notion of *continuity*, as well as *equal continuity*, as applied to a set of functions. In such a situation we are able to state the following

THEOREM. *If $\mathfrak{R}$ is a closed and compact subclass of $\mathfrak{O}$, and μ_{nr} is a monotonic sequence of functions continuous on $\mathfrak{R}$ and*

* Cf. Dini-Lüroth-Schepp: Theorie der Funktionen, p. 148.

† Cf. *Amer. Journ. of Mathematics*, vol. 34, p. 241.

‡ Cf. Fréchet: "Sur quelques points du calcul fonctionnel," *Rendiconti del Circolo Matematico di Palermo*, vol. 22 (1906), pp. 6, 7, 11.

converging to the continuous function μ_r, then the convergence is uniform and the functions μ_{nr} are equally continuous.

We prove the uniformity of convergence of μ_{nr}. Since $L_n\mu_{nr} = \mu_r$ for every r of the class $\mathfrak{R}$, we have: for every positive e there exists an n_{er} such that if $n \geqq n_{er}$ we have $|\mu_{nr} - \mu_r| \leqq e$. Suppose that we have selected for each e and r the smallest possible value as n_{er}. Then we wish to show that for each e, n_{er} is bounded on the class $\mathfrak{R}$. Suppose, if possible, this were not so for some particular e. Then for every n, there would exist an r_n, such that $n_{er_n} > n$, i. e., $|\mu_{nr_n} - \mu_{r_n}| > e$. On account of the convergence of μ_{nr} to μ_r for every r, no element can recur infinitely often in the set r_n. Then since the class $\mathfrak{R}$ is compact and closed, there will exist a subsequence $r_{n_m} = r'_m$ of r_n, and an r, such that $L_m r'_m = r$. Consider the double sequence $\mu_{nr'_m}$. It is monotonic non-decreasing in n for every m. Moreover on account of the continuity of μ_{nr} and μ_r, we have $L_m L_n \mu_{nr'_m} = L_n L_m \mu_{nr'_m}$. It therefore fulfils the conditions of our lemma, and it follows that $L_n\mu_{nr'_m}$ converges uniformly; i. e., for every positive e there will exist an n_e, independent of m, such that $|\mu_{nr'_m} - \mu_{r'_m}| \leqq e$. By taking the e as the one presupposed above, and $n > n_m$, we obtain a contradiction.

The equal continuity of the functions μ_{nr} is a direct consequence of their uniformity of convergence and continuity.

To obtain a further generalization we presuppose another general class $\mathfrak{P}$. In $\mathfrak{P}$ we shall suppose that there is defined an order relation between triplets of elements: $B_{p_1p_2p_3}$, comparable to $p_1 \leqq p_2 \leqq p_3$. We shall suppose that there exists in the class also the concept of limit, subject to the condition that, if $L_n p_n = p$, then there exists a subsequence having the same limit, such that $B_{p_{n_m}p_{n_{m+1}}p}$ for every m, or $B_{pp_{n_{m+1}}p_{n_m}}$ for every m. If μ is a real valued function on $\mathfrak{S}$, a subclass of $\mathfrak{P}$, then μ is said to be *monotonic non-decreasing* on $\mathfrak{S}$ if for every triplet s_1, s_2, s_3, of $\mathfrak{S}$ such that $B_{s_1s_2s_3}$ we have $\mu_{s_1} \leqq \mu_{s_2} \leqq \mu_{s_3}$. Finally in the composite class $\mathfrak{P}\mathfrak{D}$,* we obtain a double limit, viz., $L_{mn}p_m q_n = pq$ is equivalent to $L_m p_m = p$ and $L_n q_n = q$. This enables us to define a continuity of functions on a composite range, similar to that of continuity of functions of two variables, viz., μ is continuous on $\mathfrak{S}\mathfrak{R}$ if $L_{mn}s_m r_n = sr$ implies $L_{mn}\mu_{s_m r_n} = \mu_{sr}$. Then we have the following

* Cf. Moore: Introduction to General Analysis, p. 90.

THEOREM. *If $\mathfrak{S}$ and $\mathfrak{R}$ are closed and compact subclasses of $\mathfrak{P}$ and $\mathfrak{D}$, respectively, if further μ_{sr} is continuous on $\mathfrak{S}$ for every r and on $\mathfrak{R}$ for every s, and if moreover μ_{sr} is monotonic non-decreasing on $\mathfrak{S}$ for every r, then μ_{sr}, considered as a set of functions on $\mathfrak{R}$, are equally continuous, as well as μ_{sr} considered as a set of functions on $\mathfrak{S}$, and μ_{sr} is continuous on $\mathfrak{S}\mathfrak{R}$.*

The proof that μ_{sr}, considered as a set of functions on $\mathfrak{R}$, are equally continuous is an indirect one. The assumption that μ_{sr} is not equally continuous on $\mathfrak{R}$ is shown to be untenable by a use of the property of limit in terms of B, the monotoneity and continuity of μ_{sr}, and the preceding theorem. The details are easily supplied. The equal continuity of the set μ_{sr} on $\mathfrak{S}$, and the continuity on $\mathfrak{S}\mathfrak{R}$ follow at once from the equal continuity on $\mathfrak{R}$.

By specializing the classes $\mathfrak{P}$ and $\mathfrak{D}$, we get some interesting theorems in special fields. If we take $\mathfrak{P} = 1, 2, 3, \cdots, \infty$, with $B_{p_1p_2p_3}$ defined as $p_1 \leqq p_2 \leqq p_3$, and $\mathfrak{D}$ as the interval $0 \leqq x \leqq 1$, and note that equal continuity on $1, 2, 3, \cdots, \infty$ is uniform convergence, we get the Dini theorem stated at the outset. If $\mathfrak{P}$ is the linear interval $0 \leqq x \leqq 1$, and $B_{p_1p_2p_3}$ is the same as $p_1 \leqq p_2 \leqq p_3$, and $\mathfrak{D}$ is the set $1, 2, 3, \cdots, \infty$, we have:

*If a sequence of monotonic non-decreasing functions continuous on a closed interval converges to a continuous function, the convergence is uniform, and the set of functions are equally continuous.**

If $\mathfrak{P}$ is the linear interval $0 \leqq x \leqq 1$, with $B_{p_1p_2p_3}$ equivalent to $p_1 \leqq p_2 \leqq p_3$ and $\mathfrak{D}$ is the linear interval $0 \leqq y \leqq 1$, we have:

If $f(x, y)$, defined for $0 \leqq x \leqq 1$, $0 \leqq y \leqq 1$, is continuous in x for every y, and in y for every x, and is also monotonic non-decreasing in x for every y, then $f(x, y)$ is continuous in x and y simultaneously.

* Cf. Buchanan and Hildebrandt: *Annals of Mathematics*, ser. 2, vol. 9, p. 123. It is interesting to observe that this theorem and the Dini theorem are special cases of the same theorem.

NOTE ON REMOVABLE SINGULARITIES.

BY MR. W. E. MILNE.

IN his dissertation* Kistler proves the following theorem:

"Let $f(z_1, \cdots, z_n)$ be analytic throughout the neighborhood of a point $(a_1, \cdots, a_n)$ with the exception at most of the points of a finite number of analytic manifolds, each of which is at most $(2n-4)$-dimensional. Then it is analytic in the excepted points also, if properly defined there."

A similar theorem is given below. The two theorems differ in two respects. (1) Kistler's theorem requires the excepted points to lie on analytic manifolds, while the present one does not; (2) the present theorem requires that for *every*† pair of variables the excepted locus reduce to isolated points when the remaining $n-2$ variables are fixed, while Kistler's theorem requires this to hold simply for *one* pair. But it is to be noted that the hypotheses of the present theorem will in general‡ be fulfilled if the singular manifolds are analytic.

The theorem is as follows:

THEOREM. *Let the function $\varphi(z_1, \cdots, z_n)$ of n complex variables, $n > 2$, be analytic in the region $(S_1, \cdots, S_n)$ except for the points of a $(2n-4)$-dimensional locus of such character that when any $n-2$ of the variables are given any fixed values in their respective regions, and z_i and z_j alone vary, then the singularities occur only at isolated points (a_i, a_j) in (S_i, S_j). Under these conditions φ has a limit in the points of the singular locus, and if defined as equal to its limit, will be analytic without exception in $(S_1, \cdots, S_n)$.*

Proof: Let $z_3, \cdots, z_n$ be held fast at $(a_3, \cdots, a_n)$ any point of $(S_3, \cdots, S_n)$, and consider φ as a function of z_1 and z_2. From the hypothesis we see that singularities can occur only at isolated points of (S_1, S_2). But isolated singularities for a function of two complex variables are removable; so φ, if properly defined, will be analytic in z_1 and z_2 throughout (S_1, S_2).

* "Ueber Funktionen von mehreren komplexen Veränderlichen," Basel, 1905. § 7.

† Evidently all that is really necessary is that with each variable z_i it be possible to associate another z_j such that if the remaining $n-2$ are fixed, the singular points in z_i, z_j are isolated.

‡ I hope shortly to be able to show that the excepted cases can always be avoided by a suitable linear transformation of the independent variables, and hence that the words "in general" can be replaced by "always."

Give to z_2 any value a_2 in S_2, and φ will be analytic in z_1 alone. This holds for every choice of the fixed values assigned to $z_2, \cdots, z_n$. In a similar manner we find φ analytic in each remaining variable alone.

Now apply the theorem of Hartogs* which states that if a function of n complex variables is analytic in each one separately, it is analytic in all n variables taken together. Hence φ is analytic throughout $(S_1, \cdots, S_n)$.

HARVARD UNIVERSITY,
May, 1914.

CONCERNING A CERTAIN TOTALLY DISCONTINUOUS FUNCTION.

BY PROFESSOR K. P. WILLIAMS.

(Read before the American Mathematical Society, October 31, 1914.)

ONE of the most important properties of a continuous function is that it actually assumes every value between any two of its values. It is well known that a function can, however, possess this property without being continuous. An actual example to illustrate this seems to have been first given by Darboux in 1875. A function that is sometimes cited in this connection is due to Mansion.† The function that the latter gives actually takes all values between any two, but is discontinuous at the single point $x = 0$. Functions of this sort can be easily constructed by arbitrarily assigning the values at certain points, according to the function concept of Dirichlet. More interest would therefore attach to such a function if it is given by one and the same expression throughout its region of definition. The function given by Mansion does not, however, possess this property; for it contains the function $E(x)$, defined, as in number theory, as the integer equal to, or next smaller than x.

The purpose of this note is to give a function that takes every value between 0 and 1 inclusive, when x varies over the closed interval (0, 1), but which is discontinuous at every point. This function will, furthermore, be represented by one and the same analytical expression throughout its whole region of definition.

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* *Math. Ann.*, vol. 62 (1905), p. 1.

† "Continuité au sens analytique et continuité au sens vulgaire," in *Mathesis*, 1899.

Let $f(x)$ be equal to zero at the rational points of the interval (0, 1), and equal to 1 at the irrational points. We first obtain for this function an expression which is a modification of the one given by Hankel in his celebrated memoir on oscillating and discontinuous functions.*

Let

$$\varphi(x) = \frac{4}{\pi}\sum_{n=0}^{\infty}\frac{\sin(2n+1)\pi x}{2n+1};$$

then, as is well known,

$$\varphi(x) = \quad 1, \text{ for } \quad 0 < x < 1,$$

$$\varphi(x) = -1, \text{ for } -1 < x < 0,$$

while

$$\varphi(0) = \varphi(\pm 1) = 0.$$

This gives us for $f(x)$ the following expression:

$$f(x) = \prod_{n=1}^{\infty}\{\varphi(\sin n\pi x)\}^2.$$

The expression which Hankel gives for $f(x)$ defines it, in reality, only for the irrational points; so that its values at the rational points must be assigned.†

We next define the function $F_1(x)$ by the relation

$$F_1(x) = x + (1 - 2x)f(x).$$

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* *Math. Annalen*, vol. 20, pp. 63–112.

† Hankel puts

$$f(x) = \left(\sum_{n=1}^{\infty}\frac{1}{n^{\mu}}\right)\Big/ g(x),$$

where $\mu > 1$, and

$$g(x) = \sum_{n=1}^{\infty} 1/n^{\mu}[\varphi(\sin n\pi x)]^2.$$

He says that at the rational points $g(x)$ becomes infinite; thus making $f(x)$ equal to zero at those points. While it is true that at the rational points the denominators of some of the terms in $g(x)$ become zero, those terms do not behave in any sense as a function does at a pole. The terms abruptly take the form 1/0, and as this symbol is undefined, we cannot regard the series as defining $g(x)$ at the rational points. Other writers have given the function as Hankel gave it.

As Pringsheim has shown, we also have

$$f(x) = 1 - \lim_{m=\infty}\lim_{n=\infty}[\lim(\cos m!\pi x)^{2n}].$$

Consequently we have

$$F_1(x) = 1 - x, \text{ for } x \text{ irrational and } 0 \leqq x \leqq 1,$$

$$F_1(x) = x, \text{ for } x \text{ rational and } 0 \leqq x \leqq 1.$$

The function $F_1(x)$ accordingly takes all values between 0 and 1 inclusive when x varies over the closed interval (0, 1). It is, in addition, discontinuous at every point, save the point $x = \frac{1}{2}$. We next modify the function so that $x = \frac{1}{2}$ is also a point of discontinuity.

Let

$$\overline{\varphi}(x) = \varphi(2x);$$

then, from the above values of $\varphi(x)$, and the fact that it is periodic, we obtain

$$\overline{\varphi}(0) = \overline{\varphi}(\tfrac{1}{2}) = \overline{\varphi}(1) = 0;$$

$$\overline{\varphi}(x) = 1, \text{ for } 0 < x < \tfrac{1}{2}; \ \overline{\varphi}(x) = -1, \text{ for } \tfrac{1}{2} < x < 1.$$

Consider now the function

$$F_2(x) = \frac{(1-x)4^x}{2}\,[1 - \overline{\varphi}^2(x)] \cos 2\pi x,$$

where 4^x denotes the arithmetic root.

From the above table of values of $\varphi(x)$ we have at once

$$F_2(0) = \tfrac{1}{2}; \ F_2(x) = 0, \ 0 < x < \tfrac{1}{2}; \ F_2(\tfrac{1}{2}) = -\tfrac{1}{2},$$

$$F_2(x) = 0, \ \tfrac{1}{2} < x \leqq 1.$$

We construct finally the function

$$F(x) = F_1(x) + F_2(x).$$

It is apparent that $F(x)$ is obtained from $F_1(x)$ by merely interchanging the values at the two points $x = 0$ and $x = \frac{1}{2}$. From the properties of $F_1(x)$ it then follows that $F(x)$ takes all values between 0 and 1 inclusive, and is, furthermore, discontinuous at every point. We see, finally, that $F(x)$ can be represented by a single analytical expression throughout the interval (0, 1); for we have expressions for all the functions contained in it. We consequently have in $F(x)$ a function which possesses all the properties desired.

We shall note a few additional properties of the function we have obtained.

In addition to being single valued, $F(x)$ assumes a given value but once. We can thus regard it as giving a one-to-one transformation of the interval (0, 1) into itself, which is everywhere discontinuous. At every point save $x = \frac{1}{2}$ the function has no limit; that is, every point, except $x = \frac{1}{2}$, is a point of discontinuity of the second kind. It is also apparent that both the greatest and least values approached at a point are continuous functions.

INDIANA UNIVERSITY,
May, 1914.

PROOF OF THE CONVERGENCE OF POISSON'S INTEGRAL FOR NON-ABSOLUTELY INTEGRABLE FUNCTIONS.

BY DR. W. W. KÜSTERMANN.

IN the following pages I propose to give a proof of the

THEOREM: *If $f(x)$ is a real, periodic function, of period 2π, which in the interval $(0, 2\pi)$ has a proper or improper integral in the sense of Lebesgue, Harnack-Riemann, or Harnack-Lebesgue-Hobson,* then*

$$\lim_{r \to 1} \frac{1}{2\pi} \int_{-\pi}^{\pi} f(\alpha) \frac{1 - r^2}{1 + r^2 - 2r \cos(\alpha - x)} \, d\alpha = \lim_{t \to 0} \tfrac{1}{2}[f(x + t) + f(x - t)]$$

at every point x where the limit on the right hand side exists.

This theorem includes in particular the case where $f(x)$ remains finite—disposed of by Schwarz,† and the case where $f(x)$ becomes infinite at an infinite number of points, but has an *absolutely* convergent improper integral—discussed by Hobson and others.‡ Moreover, it goes farther, in that it

* For these definitions see Hobson, Theory of Functions of a Real Variable, Cambridge, 1907.

† Schwarz, Math. Abhd., vol. 2, pp. 144 and 175.

‡ Most completely by Hobson, Theory of Functions of a Real Variable, p. 719; cf. also Bôcher, *Ann. of Math.*, 2d ser., vol. 7, p. 81; Fatou, *Acta Math.*, vol. 30, p. 335; Picard, Traité d'Analyse, 2d ed., vol. 1, p. 268; Forsyth, Theory of Functions., 2d ed., p. 450.

also covers the case where $f(x)$ becomes infinite in such a way as to be *not absolutely* integrable in the vicinity of a point, or, for that matter, of a non-dense, closed set of points of content zero. All former proofs, as far as I know, fail to cover this case, since they require at some place or other that $|\int f(\alpha)d\alpha|$ be replaced by $\int|f(\alpha)d\alpha|$.* The subsequent proof rests upon an application of the second mean value theorem, established by Hobson† in its most general form. For the present purpose Hobson's results may be stated as follows:

"If $\varphi(x)$ be a limited and monotonically decreasing positive function of the real variable x, and if $f(x)$ have an improper integral in (a, b), either absolutely or non-absolutely convergent in accordance with the definitions of Lebesgue, Harnack-Riemann, or Harnack-Lebesgue-Hobson, then

(I) $\varphi(x)f(x)$ has also an improper integral in (a, b) in accordance with the same definition;

(II) $\int_a^b \varphi(x)f(x)dx = A\int_a^X f(x)dx$, where A is any number such that $A \geqq \varphi(a+0)$ and X is a number in the interval (a, b)."

Now consider

$$\varphi_r(\alpha) = \frac{1-r^2}{1+r^2-2r\cos\alpha},$$

where $0 < r < 1$ in the interval $0 \leqq \alpha \leqq \pi$. The $\cos\alpha$ decreases monotonically from $+1$ to -1. The expression $1+r^2-2r\cos\alpha$, however, increases from $(1-r)^2$ to $(1+r)^2$ and remains always greater than zero. It follows that, for each $0 < r < 1$, $\varphi_r(\alpha)$ is a positive, monotonically decreasing function and satisfies the premises of the mean value theorem. If then $f(x)$ does likewise, we may conclude from (I) that Poisson's integral

$$P(x, r) = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(\alpha)\frac{1-r^2}{1+r^2-2r\cos(\alpha-x)}d\alpha$$

* Thus the presence of just one singularity such as $\frac{\sin 1/\alpha}{\alpha}$ has at $\alpha = 0$ would question the existence of Poisson's integral and its limit for any point x whatever.

† Hobson, *Proc. Lond. Math. Soc.*, 2d ser., vol. 7 (1909), p. 14.

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x+\alpha)\,\frac{1-r^2}{1+r^2-2r\cos\alpha}\,d\alpha$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x+\alpha)\varphi_r(\alpha)d\alpha$$

exists and has meaning. Now

$$\int \frac{(1-r^2)d\alpha}{1+r^2-2r\cos\alpha} = \arctan\left[\frac{1+r}{1-r}\tan\frac{\alpha}{2}\right],$$

hence

$$\int_{-\pi}^{\pi}\varphi_r(\alpha)d\alpha = 2\pi,$$

independent of r. Let

$$\lim_{t\to 0}\tfrac{1}{2}[f(x+t)+f(x-t)] = L, \tag{1}$$

then

$$2\pi\{P(x,r)-L\} = \int_{-\pi}^{\pi}[f(x+\alpha)-L]\varphi_r(\alpha)d\alpha$$

$$= 2\int_0^{\pi}\left[\frac{f(x+\alpha)+f(x-\alpha)}{2}-L\right]\varphi_r(\alpha)d\alpha,$$

since $\varphi_r(\alpha) = \varphi_r(-\alpha)$,

$$= 2\left\{\int_0^{\delta}+\int_{\delta}^{\pi}\right\}\left[\frac{f(x+\alpha)+f(x-\alpha)}{2}-L\right]\varphi_r(\alpha)d\alpha.$$

An arbitrarily small $\epsilon > 0$ having been prescribed, let δ be chosen in such a way that

$$\left|\frac{f(x+\alpha)+f(x-\alpha)}{2}-L\right| < \epsilon, \text{ for } 0<\alpha<\delta.$$

That this is always possible follows from (1). Moreover, $\varphi_r(\alpha)$ being positive, $|\varphi_r(\alpha)| = \varphi_r(\alpha)$. Hence, applying the second mean value theorem (II) to $\int_{\delta}^{\pi}$, we have

$$\pi \left| P(x, r) - L \right| \leqq \epsilon \int_0^{\delta} \varphi_r(\alpha) d\alpha + \varphi_r(\delta) \cdot \left| \int_{\delta}^{X_r} \left[\frac{f(x+\alpha) + f(x-\alpha)}{2} - L \right] d\alpha \right|,$$

where $\delta \leqq X_r \leqq \pi$,

$$< \epsilon\pi + \varphi_r(\delta) G,^*$$

for $0 < r < 1$,

$$< \epsilon\pi + \frac{(1-r^2)K}{\sin^2 \frac{1}{2}\delta}$$

where G and K are positive constants,

$$< \epsilon',$$

for r close enough to unity. Hence $\lim_{r \to 1} P(x, r) = L$.

W. W. KÜSTERMANN.

ANN ARBOR,
May 2, 1914.

THE NAPIER TERCENTENARY CELEBRATION.

THE Napier Tercentenary Celebration was held at Edinburgh on July 24–28, 1914, with over three hundred visitors present. The ceremonies opened at the University of Edinburgh under the presidency of the Lord Provost of the city, the address of the day being delivered by Lord Moulton and relating to the probable reasons in Napier's mind for deciding upon a table of logarithms of sines and to his probable methods of computing. Brief addresses were given by four of the official delegates, Professors Andoyer and D'Ocagne of Paris, Smith of New York, and Bauschinger of Strassburg.

On Saturday morning the first session was held under the presidency of Professor Hobson of Cambridge. The following papers were presented:

DR. J. W. L. GLAISHER, of Cambridge: "The work of Napier."

Dr. Glaisher called attention to the paucity of notations

* The argument here is based on the fact that a definite integral is a continuous function of its upper limit and as such has a finite maximum and minimum. Cf. E. H. Moore, *Transactions*, vol. 2, pp. 296 and 459.

and general symbols three hundred years ago to aid in the forming of clear conceptions of the nature of logarithms, and stated that an examination of Napier's work impressed one with his greatness as a genuine mathematical thinker rather than as the discoverer of an isolated detail.

DR. G. VACCA, of Rome: "The first Napierian logarithm before Napier."

In the absence of the author of the paper, Dr. Knott gave a brief summary. Dr. Vacca believes that a certain passage in Pacioli (1494), in relation to the computation of interest, has in it the essence of a logarithm. The rule given by Pacioli is substantially this: Divide 72 by the rate of interest and the quotient is the required number of years that it takes a sum to double itself at compound interest. Dr. Vacca shows that this number 72 is approximately 100 log 2 in Napier's system.

PROFESSOR G. A. GIBSON, of Glasgow: "On the transition from Napier's to Briggs's logarithms."

Professor Gibson showed that the improvement suggested by Briggs in his talk with Napier was not what we commonly look upon as the Briggs logarithms. He asserted that it was Napier himself who first suggested making log 1 equal to zero.

PROFESSOR D. E. SMITH, of New York: "Laws of exponents in the works of the sixteenth century."

In this paper was discussed the gradual rise of the laws of exponents in the sixteenth century, the statements being based upon extracts from the original works of various writers. It was the author's purpose to show that the fundamental laws of logarithms were well known as applied to integral exponents, and that Bürgi was led by them to prepare his Progress Tabulen (1620). The chief purpose, however, was to bring together for the use of students the actual words of the leading sixteenth century writers relating to the subject.

LIEUTENANT SALIH MOURAD, of Constantinople: "On the introduction of logarithms into Turkey."

Lieutenant Mourad showed that Ismail Effendi (Caliph Zade) introduced logarithms into Turkey exactly one hundred years after Napier's first publication of 1614.

PROFESSOR F. CAJORI, of Colorado Springs: "Algebra in Napier's day, and alleged prior inventions of logarithms."

The paper considered the claims presented by certain writers that Regiomontanus, Wright, and Bürgi, or some one of them, had anticipated Napier in the invention of logarithms. Professor Cajori weighed the evidence in a judicial manner and showed that there was nothing to substantiate a claim to priority on the part of any of these writers.

DR. D. M. Y. SOMMERVILLE, of St. Andrews: "On Napier's rules and trigonometrically equivalent polygons, with extension to non-euclidean space."

The second meeting was held on Monday, July 27, under the presidency of Professor D. E. Smith of New York, Dr. J. W. L. Glaisher of Cambridge, and Major P. A. MacMahon of London, who served during three sessions in the order named. The following papers were read:

PROFESSOR H. ANDOYER, of Paris: "On the history and method of construction of tables."

Professor Andoyer explained the construction of his recent (1911) tables which contain the logarithms of the trigonometric functions to seventeen significant figures for every hundredth of the quadrant, and to fourteen figures for every ten seconds. He also mentioned a work upon which he is now engaged, the calculation of a table of natural sines to fifteen significant figures, a work which he expects to finish in about two years.

PROFESSOR M. D'OCAGNE: "Communications relating to the history of calculating machines and to the development of nomography."

He stated that the principle of the Millionaire calculating machine was the invention (1893) of a young French mechanician, Léon Bollée, and that the earliest systematic application of nomograms is found in Margett's Longitude Tables of 1791.

MRS. EMMA GIFFORD, of Chard, Somerset: "On a recent table of sines."

Mrs. Gifford described her method of computing and checking her recent table of natural sines, which gives the sine for every second of the quadrant.

DR. J. R. MILNE, of Edinburgh: "On the arrangement of tables."

This paper was illustrated by lantern slides, and the questions of type, workmanship, paper, and general arrangement were discussed.

MR. H. S. GAY, of Shamokin, Pa.: "On a convenient formula for determining the angle, given the sine or cosine."

Mr. Gay presented a formula which easily gives an angle to a sufficiently close degree of approximation for practical purposes, when the cosine or sine is known.

MR. J. C. FERGUSSON, of Birmingham: "On the percentage unit of measuring angles."

The essential feature is the use of the tangent instead of the angle, and of the octant instead of the quadrant.

MR. W. SCHOOLING, of London: "On the calculation of logarithms."

The speaker showed how the logarithms of numbers, the logarithms of their reciprocals, and the gaussian logarithms, can be calculated by pure addition.

DR. A. HUTCHINSON, of Cambridge: "On graphic methods and on the use of the slide rule in crystallography."

DR. W. F. SHEPPARD, of Sutton: "On the constructing of tables."

The speaker described an extension of tables by an improved method of differences.

A few other papers were, in the absence of the authors, read by title.

The social functions were well attended by all who were attracted to Edinburgh by the celebration. On Friday evening a reception was given by the Lord Provost of Edinburgh and Council, followed by a concert. On Saturday afternoon a garden party was held at Merchiston Castle, and visitors were permitted to see the room in which it is said that Napier worked out his system of logarithms. In the evening there was an informal social gathering in the Students' Union at the University. On Monday afternoon the Royal Society of Edinburgh gave a farewell reception at the Society rooms.

On Sunday a special memorial service was held at St. Giles' Cathedral, the sermon being preached by the Rev. Dr. Fisher, of St. Cuthbert's, the church in which Napier worshipped and in the churchyard of which he is buried. Dr. Fisher spoke of Napier as a citizen and as a writer on questions of theology. The service was attended by a large number of those present at the celebration.

Mention should be made of the exhibit of Napieriana and of tables and calculating machines. Early editions of works on logarithms, Napier portraits, Napier rods ("bones"), and the priceless Dantzig copy of Bürgi's Progress Tabulen were displayed in cases. All the leading types of calculating machines, models of solids and surfaces, linkages, tide predicting machinery, and other material relating to computation and to those parts of geometry in which Napier had an interest, attracted much attention.

The celebration was followed by a colloquium held under the auspices of the Edinburgh Mathematical Society. Lectures were given by Professors D'Ocagne and Whittaker, and by Messrs. E. Cunningham and H. W. Richmond.

The papers will appear in a volume under the editorship of Dr. C. G. Knott. A description of the exhibit appeared in dignified book form under the direction of Mr. E. M. Horsburgh, and a copy was presented to each participant in the celebration.

Those who attended the celebration were unanimous in their expression of pleasure at the success of the meetings and at the hospitality shown by the citizens of Edinburgh, the Royal Society, the Mathematical Society, and the University.

DAVID EUGENE SMITH.

AN APPEAL TO PRODUCING MATHEMATICIANS.

BY GEORGE PAASWELL, C.E.

A PERUSAL of modern mathematical treatises and dissertations for degrees makes the lay mathematician (I should say rather the amateur mathematician) despair of ever keeping up with the modern trend of mathematical thought. The tone of modern works is not that of disseminating new ideas, but rather that of clothing ideas already familiar to readers in

slightly different form; a tone surely not that of a text-book and therefore precluding from its readers those not familiar with the subject.

There is, however, a more serious arraignment to be made. Hardly any treatise has attempted to discuss or analyze the serious problems of the applied science professions. The profession of civil engineering is teeming with problems awaiting the solutions of a St. Venant, a Laplace, a Newton. The problems are vital; fundamental truths await discovery and elucidation. Innumerable functions await an Abel or Jacobi to bring them into existence.

Glance through the standard texts on applied mechanics and marvel at the appalling gaps in analysis the engineer must bridge with assumptions far from rigorous or satisfactory, to obtain any practical result.

The profession of building is the oldest in the world and yet how little we have progressed in the analysis of the stresses in the frame itself. The knowledge of stresses in a column remains practically in the state in which Euler left it. There are, indeed, some empirical column formulas, but an empirical formula is always a sad apology for lack of scientific knowledge. It may be said that empirical formulas have sufficed in the past for design, but the failures of structures designed on that basis most effectively answer that. The failure of the largest bridge in the world—the Quebec bridge—was due to lack of knowledge of the action of large compression members (columns) and the only path open to engineers was that of experimentation on larger size test pieces: mathematicians had failed them.

When the simple case of the analysis of flexure is attempted, there is either the faulty Bernoulli-Euler theory or else one must wade through appalling intricacies of analysis to gain the desired kernel of practical value which is then found to be limited in its application. When we come to the analysis of structures proper, there are either empirical formulas or faulty analysis. So far, I have had in mind homogeneous bodies; but when heterogeneous bodies are included, such as concrete, plain or reenforced, the ignorance of the action of the material itself adds to the sum total of our ignorance.

When we come to the analysis of statically indeterminate structures—a most fruitful field—there are several fundamental principles to be had for stress analysis; but when the

structures become less simple, the intricacies and the tediousness of analysis become inhibitive to further discussion. Yet these are the types of structures that make for economy of design and for rigidity and permanence. Here are abundant fields for mathematical thought, and every thesis and article would bring us nearer to a solution of these problems.

The subject of moving bodies with their resultant impacts and vibrations is at present hopeless—even the empirical formulas fail us here—and we must make up for lack of knowledge by excessive and wasteful strength. Likewise the study of earth pressures both vertical and lateral exists in a series of empiricisms.

There is sore need of a text on the theory of elasticity which, while more or less free of the involved intricacies, shall give a firm basis for stress and strain analysis.

In this connection the curriculums of the applied science schools are at fault. Just sufficient mathematical knowledge is disseminated to enable the student to interpret a few simple formulas and theories. No attempt is made to inculcate a rigorous and fundamental knowledge of mathematics nor to give even a hint of the vast fields of advanced mathematical thought.

The study of mathematics, per se, is a most soul satisfying and alluring pleasure, and those to whom is granted the opportunity of devoting the greater part of their time to its perusal are most sincerely to be envied; yet society asks in return a solution of its most pressing problems.

It may be visionary dreams of the wildest type, but the day may come when, from the knowledge of the chemical constituents of a body, its crystallographic history and its temperature and space elements, its stress and strain relations shall at once be known and the effects of external loadings, static or otherwise, at once interpreted.

There is hardly a branch of mathematical analysis that could not add its quota to applied science: the calculus of variations and the potential function to the principles of least work and infinitesimal distortions; the elliptic and related functions to the study of the elastica and allied problems; and general analysis itself to the reduction of the resulting expressions of stress analysis.

I have enumerated but a few of the vast problems frequently confronting engineers in daily life and fervently hope that they

may elicit some dissertations that may bring light on these vital problems.

SHORTER NOTICES.

Die Mathematik im Altertum und Mittelalter. By H. G. ZEUTHEN. Erster Abschnitt: *Entstehung und Entwicklung der Zahlen und des Rechnens.* B. G. Teubner, Leipzig und Berlin, 1912. 95 pages. Price unbound, 3 Marks.

THIS little work is a portion of Teil III of the volume entitled Die mathematischen Wissenschaften, in the work edited by Professor Hinnenberg, Die Kultur der Gegenwart. Teil III has for its title Die mathematischen und naturwissenschaftlichen Kulturgebiete, and the part relating to mathematics is under the editorial supervision of Professor Klein.

No opportunity was afforded, in the brief space allotted to Professor Zeuthen, for any new contribution to the history of mathematics. Therefore all that can be expected is a mere résumé of the leading contributions to the science in ancient and medieval times. Professor Zeuthen first treats of early arithmetic, beginning with the primitive number systems, passing to the early mechanical methods of computation, setting forth the difficulties of notation in ancient times, tracing rapidly the development of our numerals, and making clear the obstacles met by all early peoples in the handling of fractions. He then considers the applications of number to commerce, astronomy, mysticism, and puzzle problems, showing the relation of this work to the primitive algebra of the Egyptians.

He next takes up the geometry of the Egyptians and Greeks. While giving only a cursory glance at the development of the science, he takes occasion to refer to the fact that the Pythagorean triangle is mentioned in the Sulva-sutras, which he puts as late as the 5th or 4th century B.C. He does not venture, however, upon the question of the antiquity of the theorem in China, a problem which probably can be solved only by the rise of native scholars who can give us a careful textual criticism of the classics of their country. It is interesting to see that Professor Zeuthen, than whom we have few authorities better recognized in the history of Greek mathematics, maintains the common view that the lunes of Hippo-

crates are due to this writer, in spite of some doubt that has recently been cast upon it.

The summary which he gives of the work of the Greeks in geometry is, as would naturally be expected from him, very clear, and forms the principal portion of the work. The section on the early traces of the calculus is especially interesting, and refers of course to the contributions of Democritus, Eudoxus, and Archimedes, but without assigning the credit to Antiphon and Bryson that might be expected.

The mathematics of the Hindus is discussed with brief mention,—too brief, considering the contributions of the writers of India. The reader will not find the well-balanced judgment of the Oriental mathematics, nor the interesting information concerning its algebra, that he would expect from a perusal of the pages devoted to the work of the Greeks. The Arab contributions are somewhat more fully treated.

The medieval period in Europe is considered more at length, this being a period to which Professor Zeuthen has given much attention in his other works. Just at the present time, when Roger Bacon is much in the public eye, it is interesting to note that Professor Zeuthen dismisses him with exactly twelve words, and that these relate solely to optics.

It goes without saying that the book fills its purpose in a satisfactory manner. It gives a popular view of the progress of mathematics down to the period of the Renaissance, and is written in the pleasant style which characterizes all of the works of its distinguished author.

DAVID EUGENE SMITH.

Geometrie der Zahlen. Von HERMANN MINKOWSKI. Zweite Lieferung. 1910. viii + 15 pages. B. G. Teubner, Leipzig und Berlin.

THE first volume appeared in 1896. The preparation of the second volume was delayed by unexpected difficulties, and the author published in short articles in the journals most of the results initially intended for the second volume. These articles are accessible in the Gesammelte Abhandlungen of Minkowski, published in 1911 in two volumes, aggregating 872 pages. The present pamphlet is a continuation, bearing the same title, of the closing chapter of the first volume of Geometrie der Zahlen, and contains also a table of contents and index for the two volumes. This highly original contribu-

tion to the advanced part of the theory of numbers is already recognized to be of lasting importance. A more elementary introduction to the ideas there presented, as well as applications to algebraic numbers, are given in Minkowski's Diophantische Approximationen, which appeared in 1907 and was reviewed for the BULLETIN by the present writer, February, 1909 (volume 15, pages 251–252).

L. E. DICKSON.

Algebra of Quantics. By E. B. ELLIOTT. Second edition. Clarendon Press, Oxford, England, 1913. xvi + 416 pp.

THIS new edition of Professor Elliott's well-known work on algebraic theory presents more changes in typography than in content or method. Although the material has been considerably increased (by one eighth is the author's liberal estimate), the number of pages has been actually decreased. This has been accomplished by more compact printing, the adoption of a new method of writing fractions, and the insertion of many important equations in the body of the text instead of printing them in separate lines. These changes do not add to the appearance of the text, and surely not to the delight of the reader, to whom the outstanding equations were of great assistance in reference. The change from d to δ in writing partial derivatives strikes the eye at once. Paragraph numbers are practically unchanged.

Conservatism marks this new edition. Professor Elliott has not abandoned the English methods for German symbolism. In a majority of paragraphs there is no change, and most of the others have only slight verbal changes or an added sentence to clear up obscurities or to suggest further deductions. Chapters V and XV alone present important modifications. In Chapter V, on binary quantics, there is considerable rearrangement, and material has been added on "Invariants as functions of the differences of roots." A half dozen added pages are devoted largely to establishing the conditions under which $F(a_1, a_2, \cdots, a_p)$ can be expressed as a function of the differences of the arguments and to proving some properties of this function. The author notes that this should have been in the first edition. Chapter XV, on restricted substitutions, has undergone the most changes and contains the most new material. The half dozen paragraphs on Boolean systems for the linear form, the quadratic, the

linear and quadratic, etc., have been replaced by one short chapter which suggests the method by which the Boolean systems for quantics of low order may be calculated. Ten pages have been added on the deserving topic of orthogonal concomitants. Both the absolute and non-absolute concomitants are treated for direct and skew orthogonal transformations. Complete systems of orthogonal invariants and covariants are given for the quadratic, cubic, and quartic. Much of this theory has been developed since the first edition, although some of the theorems date back to Sylvester.

Some other changes may be worth noting. In Chapter IV a few pages on German symbolism are added and the relation between it and hyperdeterminants is shown. In Chapter VII, dealing with the operators O and Ω, a long paragraph is inserted on "Separation of gradients into seminvariants and other parts," with examples. In Chapter VIII, on generating functions, an important article is added on a method of extracting generating functions for *invariants* only, and is illustrated by the sextic case. The real generating function for the sextic has been added to the table. In Chapter IX, we find a new paragraph on Hilbert's second proof of Gordan's theorem and a short one on syzygies and syzygants. Chapter XI contains a new proof of the theorem that every G is an ΩF, where G and F are both gradients, covering the case where F contains more advanced letters than G. The possible nature of F is brought out clearly in the discussion. Chapter XII contains a new article on apolarity and another on the canonical form $(1, 0, C, D, E, 0, 1)\ (X, Y)^6$ of the sextic. In Chapter XIV, the concomitants of the linear form and a p-ic are given and the most important of several additions in Chapter XVI is on "A generator of all covariant ternary sources."

The above with various minor additions contain scarcely enough of novelty to warrant the entry of this second edition upon a shelf already containing the first, although the added examples are of great value. In the preface the author expresses the hope that the work will continue to appeal to the beginner and further "trusts that the book will retain for some time to come a certain value to those . . .who will need to look back upon and utilize the labors of those pioneers of modern algebraic theory, who adorned the latter half of the nineteenth century, and have now passed away." The demand for a second edition makes it reasonable to expect that this hope will be realized. D. D. Leib.

Problèmes d'Analyse mathématique. By E. FABRY. Paris, Libraire Scientifique A. Hermann et Fils, 1913. 460 pp.

THE primary object of this collection of problems seems to be to furnish material in which French student candidates preparing for the examinations for certificates in differential and integral calculus might be interested. A list of problems (279) given at examinations within the last twelve years and at fourteen different examination centers scattered throughout France occupies the first 66 pages of the book. The detailed solutions of the problems proposed fill quite completely the remainder of the volume. The reviewer has checked a few—not many—of these solutions.

The 279 problems in the twelve chapters are distributed in numbers from 5 to 35 over the field of analysis as follows: Two chapters on quadratures and their geometric applications. One on line integrals (complex variable, analytic functions). Four on differential equations and their applications to plane curves, space curves, and surfaces, including the determination of geodesics, asymptotic lines, and lines of curvature. A chapter on ruled surfaces (developable). Two chapters on partial differential equations and their geometric applications. One each on total differentials and elliptic functions.

Some of the problems—a few—might be included in the lists of "harder problems" in calculus texts designed for the American college sophomore; more could be solved with much labor in algebraic and trigonometric reductions by a class in the advanced calculus, but many would undoubtedly have to be "left for later consideration." Certainly, if the candidate passed an examination on a similar list he might well feel that he deserved his certificate.

ERNEST W. PONZER.

Cours de Géométrie infinitésimale. By G. DEMARTRES. Paris, Libraire Gauthier-Villars, 1913. x + 455 pp.

EVEN a cursory inspection of this volume would lead one to conclude that it was never intended to rival the classic works of Bianchi or Darboux. One easily comes to the conclusion, especially after reading the excellent preface by Appell, that the book is intended to be a text, not a reference book to accompany lectures, on a subject which, according to the reviewer's opinion, is sadly neglected in mathematical

courses offered in this country. A course on differential geometry after the plan of the book under review or that of Scheffer's "Anwendung der Differential und Integral-rechnung auf Geometrie," with which one is continually tempted to make comparisons, would help round out any undergraduate course in mathematics offered. Incidentally it would serve as a safe stepping-stone from the calculus to the maze of analysis into which the student too often is plunged headlong.

The correct pedagogical principle, that by far the majority of students feel constrained to make use of geometric interpretations of analytic processes in order to understand the latter thoroughly, is kept constantly in the foreground. In fact it is with this purpose in mind that the whole of the first part of the book (134 pages) is developed along entirely geometric lines. Here the geometry of the infinitesimal by no means plays the minor rôle it too often is assigned, if it is not neglected entirely—yes, even in treatises on geometry! The theory of infinitesimals and its application to the problems of curvature, order of contact, special curves, kinematics, etc., is developed geometrically. So also for space curves and surfaces. Definite notions are given concerning the nature of curvature, principal normal, tangent plane, osculating sphere, asymptotic and geodesic lines, etc., and theorems are proved entirely from the geometric side.

If it should be that the analysis because of its generality, rigor, and art of being conclusive appeals to the student with greater force, then he might well begin the book with the second part; for the last three of the four parts into which it is divided emphasize the analysis. Or he could compare the analytic treatment of the subject with the geometric given in Part I—a comparison which would certainly help fix its principles. However, as far as possible, an endeavor is made to hold fast to geometric interpretation no matter how analytical the treatment becomes. As an aid in this direction may be mentioned the general plan which is carried throughout the last three parts. This consists in expressing the coordinates of a moving point by a power series in terms of the displacement along a curve. A proper choice of the axes at the origin of the moving point allows a geometric interpretation of much of the analysis at each instant in terms of simple geometric relations.

The theory of lines, straight or curved, in a plane, free to move, or situated on a surface, together with the usual and some

special theorems accompanying it, is developed in Parts II and III. In Part IV curvilinear coordinates are used in the problems and theorems of space curves and surfaces. Here is also included the application of line coordinates to the general theory of surfaces of the second order.

A list of 136 exercises, to some of which is added the suggestion that they be solved geometrically, is given at the close of the book.

It is to be regretted that the volume does not possess the high typographical tone and finish which characterize Scheffer's text.

ERNEST W. PONZER.

Untersuchungen über die im Schlusswort des Lie'schen Werkes "Geometrie der Berührungstransformationen" angedeuteten Probleme. Von PHILIPP ENGELHARDT (Inaugural-Dissertation, Würzburg). Leipzig, B. G. Teubner, 1910. 65 pp.

THE closing lines of Lie's Geometrie der Berührungstransformationen forecasted that a second volume, which never appeared, would treat the theory of differential equations from the standpoint of contact transformations and continuous groups. One may well hope that this field will soon receive its fair share of attention and be extensively developed beyond the elements established by Lie.

The thesis under review might be classed with fundamental literature having this tendency. The author's methods are, for the most part, those presented by Lie in chapter 14. The integral surfaces of a non-linear partial differential equation of the first order, $F(x, y, z, p, q) = 0$, bear an aggregate of ∞^3 exceptional curves called characteristics (Geometrie der Berührungstransformationen, pages 261, 498). Lie's summary of his last chapter is in the form of statements of six problems which are there solved. These are determinations of differential equations $F = 0$ defined by definite conditions imposed upon the characteristics or upon the normals to integral surfaces. Problem 2 is the determination of the equations $F = 0$ whose characteristics are lines of curvature upon all integral surfaces. Problem 4 is the determination of the equations $F = 0$ whose integral surfaces have for normals precisely lines of a given line complex. This thesis is devoted to twelve problems obtained by combining Lie's six problems. Thus problem (2, 4) of the thesis is the deter-

mination of all equations $F = 0$ whose characteristics form lines of curvature upon all integral surfaces and whose integral surfaces have for normals precisely lines of a given line complex. These prove to be equations having integral surfaces for which the complex of normals is the ∞^2 rays through a definite curve k. The characteristics are circles.

Geometrical intuition based upon configurations defined by Monge equations gives direction to the analysis. There is no introductory summary of the principles, although a desirable outline could have been given in a comparatively brief space and would have improved the exposition.

O. E. GLENN.

The Trisection of the Angle by Plane Geometry: Verified by Trigonometry with Concrete Examples. By JAMES WHITEFORD, B.A., M.D. Bowes and Bowes, 1911. 169 pp.

THE book under review is another illustration of how little the logic involved in the impossible construction proofs of Klein and others is appreciated by the ordinary mind. While actually quoting DeMorgan to the effect that the famous trisecting of the angle problem cannot be solved, yet the author claims to present a right line and circle solution of this famous problem. His confidence in his construction and proofs seems to rest upon the fact that they stand the test of trigonometry aided by logarithms in fifty concrete examples laboriously calculated to seven places of decimals. It is hardly necessary to say that he has not solved the famous trisection problem but simply exhibits a rather close approximation method. His construction is not even useful as a practical drawing-room approximation since simpler and more exact angle trisection methods are well known, although of course not euclidean methods. What a pity that so much time, patience, and labor should be wasted in such useless work!

ERNEST B. LYTLE.

Practical Algebra, Second Course. By J. V. COLLINS. American Book Company, 1912. x + 303 pp.

THIS book is intended to meet the needs of a text for a second course in algebra in such schools as offer a second course, often elective, in either the junior or senior year. After a rather rapid review of the usual topics of a first course

in algebra, follow chapters on ratio and proportion, logarithms, arithmetical and geometrical progressions, the binomial theorem, inequalities, and exercises in physics.

The style is simple and the problems are numerous; the function notion is introduced and used in eight different places; the sine, cosine, tangent and cotangent functions are used in the solution of right triangles; correlation with geometry is made through problems involving geometric principles; graphic methods are used frequently; proper emphasis is placed upon translation from English to algebra and from algebra to English; historical notes and pictures of famous mathematicians appear in many places.

While the book as a whole seems to be a satisfactory text, yet we find a few points to criticise. Since ∞ is not a number symbol in elementary algebra, we dislike "The symbol ∞ denotes an exceedingly large number," etc., page 4. "The exponent 0 shows that x is used no times as a factor of the product, or has dropped out and so does not affect the product of the other factors," page 146, seems an unfortunate expression and not helpful in emphasizing the important fact that any number with a zero exponent is equal to unity. In the treatment of imaginaries it is feared that the student will miss the importance of expressing in the i-form before attempting operations with imaginaries. In variation the almost obsolete notation is used which many wish to see eliminated; the better equality form is mentioned but not sufficiently emphasized to avoid the difficulties students usually have with the language of variation. In the theory of the quadratic the general form $ax^2 + bx + c = 0$ seems much better and is more usual than the form $x^2 + px + q = 0$ which the author uses; $p^2 - 4q$ is certainly not the usual or common form of the discriminant of the quadratic.

ERNEST B. LYTLE.

Complete Business Arithmetic. By GEORGE H. VAN TUYL. New York, American Book Company, 1911. 4 + 416 pp.

ONE of the most serious problems which we have to face in the teaching of elementary mathematics at the present time relates to the work in our commercial courses. The world is far from coming to a decision as to what mathematics is best suited to the training of the boy and girl who propose to enter the field of commercial activity. The school of educators

who believe in nothing but the immediately practical would admit only arithmetic, while those who believe that the person who is to succeed in any line of activity must have come in contact with all of the great fields of human interest, and in particular with mathematics, will demand at least some slight knowledge of algebra, geometry, and trigonometry.

In this country our schools of commerce have emphasized the work in arithmetic, rarely paying any attention to the adapting of algebra to their purposes, or giving any consideration to the important field of measurement. In this we are distinctly behind the schools of Germany, Switzerland, and Austria, to name the three countries which have given the most attention to the subject.

The work of Mr. Van Tuyl is typical of what we are doing in this country, and if it is not up to the ideal standard the blame is, to a great extent, due to the fact that we have not seriously attacked the problem of the curriculum. It seems very strange that a student who is using a book of this kind should need to be drilled upon the combinations in addition (page 31), or to have the simple process of multiplication explained to him (page 52), especially as the more difficult process of ordinary division is not explained at all. It is difficult to see why a student in commercial arithmetic should be spending his time in finding the prime factors of 12,464 (page 61), the greatest common divisor of 6,859, 11,191, and 3,610 (page 63), or the least common multiple of 36, 45, 72, 105, and 150 (page 64). In this era of the decimal fraction and of approximations, it is hard to justify the reduction of $\frac{3125}{}$ to an integer (page 72), or $1846\frac{3}{7}$ to an improper fraction (page 73). It would also trouble us to say what commercial interest is touched by such problems as the simplification of $\frac{7}{15}$ of $22\frac{1}{2} \div \frac{8}{13}$ of $\frac{24}{33}$. Why a subject like compound proportion (page 116), or even simple proportion as a method of solving commercial problems, and particularly with the old symbol (: :), should have place in a modern book like this it would be hard to tell, and it would be even more difficult to justify the expenditure of time necessary for dividing 8Kl. 7Hl. 5Dl. 7l. 6dl. 4cl. 5ml. by 5 (page 149).

On the other hand, the drill on the checking of computations, the work in graphs, and a considerable part of the work in elementary business principles have much to commend them.

One lays down the book with the feeling that our commercial

courses in this country have not advanced as rapidly as our ordinary curriculum, and that, in spite of the good features mentioned above, this work will not help to improve the unfortunate situation.

DAVID EUGENE SMITH.

Text-book of Mechanics. By LOUIS A. MARTIN, JR. Vol. III: *Mechanics of Materials*, 1911, xiii + 229 pp. Vol. IV: *Applied Statics*, 1912, xii + 198 pp. 12mo. New York, John Wiley and Sons.

THE first two volumes of this text* deal with theoretical mechanics. The volumes under review, together with a projected volume on applied kinetics, will constitute a course in applied mechanics.

Volume III presents the usual theory of the deflection of beams by simple or compound stresses, statically indeterminate beams, struts and columns (eccentric loading, buckling, etc.), elastic failure, and envelopes. Also the principle that the external work done by the applied forces must equal the total resilience due to bending and shear is extensively used to determine deflections; and the equations of several elastic curves are very neatly found in this way. There is no treatment of curved bars, flat plates, or rotating disks, nor any mention of core sections in eccentrically loaded columns, but the problem of avoiding tension in such columns is treated briefly.

A fairly extensive range of applications is covered in the problems; but one is rather surprised at the omission of any applications to the distribution of steel in large reenforced concrete tanks, inasmuch as so many of these are now in use, at least in the west. Moreover, it might be well to include in a later edition a comparison of the various formulas for long columns as regards typical results, and also to insert a convenient collection of the most important tables and formulas. This would not hurt the book as a text and would make it more attractive for reference purposes.

The author says in the preface that he has attempted "to produce a book which will encourage the student to think and not to memorize, to do and not simply to accept something already done for him; but which still furnishes sufficient material in the way of explanation and example so that he

* Reviewed in this BULLETIN, vol. 16 (1909), pp. 144–7.

will not become discouraged." He has succeeded remarkably well. There is no tendency toward "spoon-feeding"; yet the treatment is admirably clear and simple. Principles are concisely stated; and, with scarcely any exceptions, the exposition has been carefully thought out.

It might be well to clarify the expression, "the inscribed curve to this polygon" (page 45), to explain why stresses may be regarded as constant over the surface of an infinitesimal element on page 137, while their increments had to be considered on page 135; to comment on the two apparently distinct meanings of the symbol $'M_x$ on pages 188–9; and to make the resilience formula (page 15) more general by changing the limits of integration to Δl_1 and Δl_2. Perhaps it would also be well, before asking the student to derive the equations for the elastic curve of a certain continuous beam (example 106), to suggest in some way the device of equating slopes of adjacent spans at an abutment to help determine the constants of integration.

Naturally mathematicians will object to speaking (pages 26, 32) of a discontinuity in a function or its graph at a point where the slope has a discontinuity but the graph itself is free from a break; also to the statement (concerning the vanishing of Q whenever M reaches a maximum) "as must evidently be the case from the relation $dM/dx = Q$." But let us refrain from throwing this last stone, until all of our standard texts on calculus have eliminated similar statements!

Of course, calculus is freely employed in this volume; and the method used in formulating differential relations is the usual one of considering an infinitesimal element and regarding some variable quantity as constant throughout the small element, and often incidentally dropping infinitesimals of higher orders. (To cite a simple case from volume IV, the differential of work, $dW = pdv$, is obtained by regarding p as a constant during the small change in v.) In teaching calculus to beginners this method is wholly unsatisfactory; but in making applications of the calculus it is far more convenient than any argument concerning limits, and is in almost universal use by practical men. Judging by our texts on calculus, is there not a gap right here in our teaching of the subject? Would it not be well toward the end of the calculus course to treat thoroughly this matter of formulating differential relations, with a detailed comparison of the two methods (which have

been aptly called the rigorous and the vigorous), and with considerable practice on real applications? Possibly the present inadequate treatment of this matter is one cause of the difficulty which students generally have in connecting their calculus with their practical work.

But to return. Few errata have been noticed, and in most cases the correction is fairly obvious: read x^3 for x^2 (page 36, line 13); $\bar{y}$ for y (page 66, line 20); $\frac{\pi}{4}+\frac{\phi}{2}$ for $\frac{\pi}{2}+\frac{\phi}{2}$ in fig. 83 (page 160); *section* for *material* (page 177, line 23); *uniformly* for *gradually* (page 36, etc.); e for c in the answers to examples 113–116; and interchange t_1 and t_2 in the answer to example 119.

Volume IV reviews and supplements numerous sections in Volumes I, II; and carefully summarizes the theoretical mechanics presupposed for the applied work. The applications include dynamometers, expansion of steam, tractive power of locomotives, abutments, cranes, steam shovel and dipper dredge, cables, wedges, keys and cotters, sliding arms, screws, pivots, belts, journal friction, resistance to rolling, etc.

The treatment of solid statics presupposes no knowledge of the analytic geometry of space beyond the meaning of the three coordinates. (In fact it is surprising how little analytic geometry of the plane is used in many branches of applied mathematics, even where the calculus is freely employed. If we are interested in letting our students get early those tools which they will most use, it would seem desirable for us to postpone much of the analytic geometry, letting it follow an introductory course in calculus, and thus make room for early treatment of the latter subject.)

The author bases his entire treatment of the expansion of steam upon the assumption that Boyle's law applies; but he treats adiabatic expansion in the case of air. It might be well at least to remark that the indicator diagrams for steam engines often satisfy a law of the adiabatic type better than Boyle's law.

Methods of approximating plane areas are discussed; and Simpson's rules are given in their usual extended forms, rather troublesome to remember. Why not modify the statements, making the basis of each rule a formula for the area of a single strip,—e. g., $A = \frac{1}{3}(y_1 + 3y_2 + 3y_3 + y_4)l$ in the case of the three-eighths rule, where y_1 and y_4 are the bounding ordinates

of the strip, and y_2 and y_3 are the ordinates trisecting the base (whose length is l). The general rule is, then, simply to divide the area into any number of strips and apply the above formula to each strip. Why bother to remember the result obtained by summing, and why base each rule upon a set of several strips and then have to remember whether the rule requires the total number of strips to be a multiple of two or three?

But this criticism, in common with several above, is directed more at current practice than at the particular volumes under review. In fact, the text is one of conspicuous merit, which not only promises to give good results, but also gives every indication of representing a course already taught with a high degree of success.

F. L. GRIFFIN.

Die angewandte Mathematik an den Deutschen mittleren Fachschulen der Maschinenindustrie. Von Dipl.-Ing. KARL OTT. Band IV, Heft 2, Internationale Mathematische Unterrichts-Kommission. Leipzig, Teubner, 1913. vi + 158 pp.

CONFESSIONS of faith are much in vogue these days with politicians, all of whom desire to be considered progressive. The author of the pamphlet under review uses considerable space, especially at the beginning, to create the proper atmosphere around the definite points of view from which the work of the particular schools on which he is reporting is to be judged. And in the confession of the principles of his faith he uses the words of Runge, of Göttingen, to express himself. Freely translated and condensed, these principles are about as given below.

The problems of applied science requiring mathematics call for quantitative results. Abstract and formal results are not sufficient. They must be correct to the proper decimal place. They should check. Accuracy and efficiency must obtain. Form and logical sequence must be seriously considered. . . . Suitable methods whether graphical, numerical, or mechanical are necessary. The province of applied mathematics is to furnish such suitable methods. And the field is not at all to be considered as apart from pure mathematics but a part of it. It is simply a question of a different state of mind handling its problems efficiently. To all of which the author adds in his own words that the engineering student must have a

thorough understanding of the fundamental principles of the mathematics he applies and be able to reason clearly along technical lines. However, he need not go into the full theory (abstract) of all the principles. An engineer may not be familiar with the writings of Carnot or Clausius, but he should be familiar with the thermodynamic principles they discussed and have had much experience in their application. Empirical methods and results arise and are well worth while. . . .

The schools under consideration arose because technical industry required them. They are strongly influenced by actual engineering practice, and of late years the Verein Deutscher Ingenieure has helped outline their programs and general engineering training.

The province of this special report is limited to those studies in a technical course in which the mathematics involved forms the back-bone. These are technical mechanics, including mechanics of materials, graphic statics, hydraulics, machine design, thermodynamics, and a general study of the problems of electrical engineering. The following is a composite program in a typical government school of the type under discussion.

First semester: Statics. Technical mechanics, with applications to mechanics of materials. Elements of graphic statics. 6 hours per week.

Second semester: Dynamics of rigid bodies. 6 hours per week.

Third semester: Hydraulics. Thermodynamics. 6 hours per week.

The methods of the seminar and laboratory play an important rôle in the instruction, while design as handled in this country meets with the author's warm approval.

Students enter differently prepared. The cry about the inability of some of their students to handle the fundamental principles of trigonometry, powers and roots, quadratics, logarithms, etc., sounds very much like the tale of woe uttered by our own instructors of freshman mathematics. Some schools require for entrance the equivalent of our customary first year mathematics, others offer a preparatory course. It takes a year, as with us, to obtain an efficient homogeneity of ability and preparation.

Some of the schools carry pure mathematics (analysis) and the applied mathematics at the same time; some try to cor-

relate; others try the spiral method, where the principles are reviewed in the light of the principles of the calculus. With others the calculus is not studied as a separate subject and mechanics is handled by laboratory methods. In general the calculus is studied later than is the practice in our country and the whole plan of procedure is not outlined as definitely as with us. The case is worse in the private schools of similar kind. Undoubtedly the situation calls for instructors of ability to handle the applied mathematics—or to side-step its problems with agility.

The subject matter during the first half year must necessarily be quite elementary, and there are evidences of ingenious ways of dodging the rigors of the calculus. On the other hand there are equally ingenious demonstrations of a "near calculus" which in the nature of things must always arise in actual engineering problems. Definitions, approximations, experiments, mensurations, rules, formulas, instruments, models—all are used. Most excellent aids—all of them; but should they be first aids?

Examinations are given at the end of the course. These are partly oral, partly written. They are in charge of special committees of school officials and other dignitaries, including the instructors. The typical examination papers shown seem to be not at all as difficult as the average semester examinations on similar work in our technical schools.

A chapter on texts in use, their strong and weak points, is given. A list of reference works and journals is included. Slide rules, planimeters and models used in teaching are discussed. Typical methods of handling fundamental principles naturally calling for the calculus without its aid are illustrated. Graphical methods with special reference to graphic statics receive much encouragement.

Descriptive geometry here, as in our technical schools, aims to develop the students' power to see space relations. The curriculum seems not to include the mathematical treatment of the subject which so many critics have declared desirable. ERNEST W. PONZER.

Astronomy, A Popular Handbook. By HAROLD JACOBY. The Macmillan Company. xi + 435 pages.

THIS book has a broader purpose than its title would seem to indicate. It is intended to be suitable not only for the

general reader, but also for elementary college classes. As the author himself points out in the preface, these two purposes appear to be contradictory. While the general reader requires a treatment devoid of all mathematics, the college student should gain the additional insight into the subject made possible by the occasional use of algebra and geometry. The plan by which the author has met the difficulty is simple and natural. He has made the body of the text free of all mathematics, and has placed in an appendix such algebra and geometry as is used. Throughout the book it is apparent that he has aimed to make the presentation easy and attractive. The mathematical notes themselves are not at all difficult, and could certainly be mastered by any college student.

Although the author's clear and easy style of writing has enabled him to produce a book that is indeed interesting, it seems to me that upon certain questions he has given undue prominence to his own opinions, with a marked exclusion of many important facts. This is particularly unfortunate in a book intended largely for the general reader. The layman is apt to take the writer's words as those of an authority, and he, furthermore, has no way to know that he has read only a small side of a question. I refer to Professor Jacoby's treatment of the markings on Mars. In the discussion of this subject the book is in marked contrast to some of the excellent texts in general use, in which the strongest evidence in favor of, as well as against the canals, is plainly stated, and in which special emphasis is not placed on the writer's views. That Professor Jacoby's ideas concerning the canals are decided, and are emphatically expressed, is shown by the following statement, page 231: "We conclude that neither by visual nor by photographic evidence has the existence of an artificial network of markings been proven, or even rendered highly probable. Therefore the time has not yet come when we shall have to inquire whether geometric lines indicate the presence of intelligent inhabitants; that time will arrive if the lines themselves are ever shown to possess a real or even a highly probable existence." I do not think that I misunderstand the writer. It is true that the first sentence contains the word *artificial*; but the final statement leaves no doubt that it is the existence, not the origin of the lines, that the author is aiming at.

Professor Jacoby's treatment of the Martian question, and

the character of the arguments that he advances, have already been considered at length by Mr. E. C. Slipher.* Mr. Slipher is an experienced observer, and has studied the planet under the most excellent conditions. I refer the reader to his paper for a fuller discussion than is given here. In view of the weight that might attach to a book by Professor Jacoby, it seems to me proper, however, to call attention in this place to some of the opinions that he advances.

It would seem that Professor Jacoby is an ultra-sceptic on the Martian question. As Mr. Slipher points out, he doubts even the universally accepted behavior of the polar caps. We read, page 225: "The polar spots *seem* to increase in the Martian winter season, and to diminish in the summer. *If so*, they may be ice caps, etc."† Small wonder then that he refuses to accept the existence of canals if there still lurks with him doubt concerning the behavior of the polar spots. If the agreement of observers since the time of Herschel is not sufficient to allow him to hold a more positive opinion regarding the caps, it is hard to imagine what form of proof he would require of the fainter markings.

As is the case with all those that deny the existence of canals, Professor Jacoby defends his position by advancing an illusion theory. In so doing he states certain views to which I do not think that observers, or scientists in general, will subscribe. On page 229 we find the following statements regarding the effect of training and experience in observing: "This theory explains why highly experienced observers see so much more than beginners. They think they are training the eye, so as to increase its powers, while in reality they may only be training that slight imperfection of the imagination which tends to increase details thought to be visible. . . .Nothing more strongly increases the powers of the imagination—of seeing the unseen—than the knowledge that others have already made the observation. We are very prone to 'see' what we are told by others is visible: we think we see what we desire and hope to see; do what we will, we cannot prevent this." It is not by advancing such theories that the Martian question will be settled. The decision lies with those who have observed the planet continuously under the best conditions that can be found; and the testimony of most of these is in strong

* "The Martian markings," in *Popular Astronomy*, March, 1914.
† The italics are mine.

agreement, both as to the character and extent of the markings. I imagine that Professor Jacoby regards all the details that he himself sees on Mars as real, and not the product of his imagination, even though some of them might not be seen by others, less experienced than himself.

It seems to me very unfortunate that statements, such as those quoted above, should be made in a popular book by a professor of astronomy. They are in no way calculated to increase the confidence of the public in the value of astronomic study. Perhaps the writer did not mean them to be as far-reaching as they seem. Doubtless he merely overstated himself in a moment of temporary enthusiasm on the Martian question; for we find a few pages later the following quotation from Herschel: "I determined to accept nothing on faith, but to see with my own eyes what others had seen before me." Or was this quotation given to discredit Herschel's work?—plainly he had the avowed purpose to "see" what he was "told by others was visible."

I shall mention an instance in which the author puts particular value in negative evidence. Professor Campbell's observations are taken as conclusive that water vapor does not exist on Mars. Why make no mention even of the fact that Mr. Very's measures of Dr. V. M. Slipher's spectrograms, without exception, show that water vapor is present? They were made with the best of instruments and under the most favorable conditions.

It is significant that the book contains no copy of any of the many beautiful drawings of Mars that show the canals. Was it not safe to gratify the natural curiosity of the reader, and allow him to see how so many have pictured those strange products of their imagination?

K. P. WILLIAMS.

NOTES.

THE annual meeting of the American Mathematical Society will be held in New York City on January 1–2, 1915. At this meeting President E. B. VAN VLECK will deliver his Presidential Address, on "The role of the point set theory in geometry and dynamics." The Chicago meeting of the Society will be held on December 28–29. Section A of the

American association for the advancement of science will hold its sessions on December 30–31, at Philadelphia.

A NEW edition of the List of Officers and Members of the American Mathematical Society is now in preparation, and will be issued in January. Blanks for furnishing information have been sent to members. A prompt response is requested, in order that the data may be correct and complete.

THE office of the American Mathematical Society, provided and partly furnished a year ago by Columbia University, was destroyed by fire on the early morning of October 10, with complete loss of files and records and a considerable stock of volumes of the BULLETIN and *Transactions*. Of the *Transactions* and the later volumes of the BULLETIN, the Society still has a sufficient supply. But the first ten volumes of the BULLETIN were completely destroyed. The Society will be glad to receive gifts of any of these early BULLETINS, and also copies of the Annual Register and Catalogue of the Library, which were also lost in the fire.

THE concluding (October) number of volume 15 of the *Transactions of the American Mathematical Society* contains the following papers: "The conic as a space element," by R. A. JOHNSON; "The Weierstrass *E*-function for problems of the calculus of variations in space," by G. A. BLISS; "The subgroups of the quaternary abelian linear group," by H. H. MITCHELL; "Transformations of conjugate systems with equal point invariants," by L. P. EISENHART; "Proof of the finiteness of the modular covariants of a system of binary forms and cogredient points," by F. B. WILEY; "On the degree of convergence of Sturm-Liouville series," by DUNHAM JACKSON; "Singular integral equations of the Volterra type," by C. E. LOVE; "On the reduction of integro-differential equations," by G. C. EVANS; "Invariants in the theory of numbers," by L. E. DICKSON. Also addenda and errata for volumes 11 and 14 by E. B. LYTLE and OSWALD VEBLEN, and general index of volumes 11–15.

THE concluding (October) number of volume 36 of the *American Journal of Mathematics* contains: "The quartic curve and its inscribed configurations," by H. BATEMAN; "On the continuity of a Lebesgue integral with respect to a

parameter," by J. K. LAMOND; "Geometry on ruled surfaces," by S. LEFSCHETZ; "Restricted systems of equations (second paper)," by A. B. COBLE; "Character of the solutions of certain functional equations," by T. E. MASON; "Binary conditions for double and triple points on a cubic," by L. A. HOWLAND; "Modular invariants of two pairs of cogredient variables," by W. C. KRATHWOHL.

THE opening (September) number of volume 16 of the *Annals of Mathematics* contains: "Invariantive characterization of some linear associative algebras," by OLIVE C. HAZLETT; "On the theory of n-lines," by LENNIE P. COPELAND; "Plane curves with consecutive double points," by F. R. SHARPE and C. F. CRAIG; "The effect of radiation on a small particle revolving about Jupiter," by T. H. BROWN; "Non-homogeneous linear equations in infinitely many unknowns," by ANNA J. PELL; "The inscribed and circumscribed squares of a quadrilateral and their significance in kinematic geometry," by C. M. HEBBERT; "A note on symmetric matrices," by G. A. BLISS.

AFTER ten years of successful experience, the Mathematical club of Syracuse University has been reorganized into a mathematical fraternity, Pi Mu Epsilon, whose aims are the advancement of mathematics and scholarship. The fraternity is incorporated under the laws of the State of New York, with power to grant charters to other chapters.

THE Association of mathematics teachers of New Jersey held its first meeting at Rutgers College on November 7. The programme included an address of welcome by President W. H. S. DEMAREST of Rutgers College and response by H. E. WEBB, and the following papers: "The place of mathematics in the high school curriculum," by G. H. ECKELS; "Number and the quadratic," by RICHARD MORRIS; "The mechanics of aviation," by L. P. EISENHART; "Advanced mathematics in the high school," by R. W. LORD; "High school mathematics," by T. G. VAN KIRK; "Fractional equations in algebra," by H. H. ALLEN; "Certain definitions in geometry," by H. E. WEBB. These were followed by the report of the committee on the constitution. The temporary officers of the society are RICHARD MORRIS, president; H. H. ALLEN,

secretary-treasurer; G. G. BROWER and H. E. WEBB, executive committee.

UNIVERSITY OF MUNICH.—The following mathematical courses are announced for the present semester: By Professor F. LINDEMANN: Theory of functions of a complex variable, four hours; Plane analytic geometry, four hours; Theory of higher algebraic curves, two hours; Seminar, two hours.—By Professor A. VOSS: Differential calculus, four hours; with exercises, two hours; Mechanics, four hours; Seminar in mechanics, two hours.—By Professor A. PRINGSHEIM: Algebra, four hours; Theory of numbers, four hours.—By Professor H. BRUNN: Elements of higher mathematics, four hours.—By Professor F. HARTOGS: Descriptive geometry, I, four hours; with exercises, three hours; Theory of determinants, two hours.—By Dr. F. BÖHM: Integral calculus, with exercises, five hours; Mathematical statistics, four hours; Mathematical literature, two hours.—By Dr. H. DINGLER: Elementary mathematics, four hours; Differential geometry of plane curves, two hours; Colloquium, two hours; Reduction of observational data, two hours.—By Dr. A. ROSENTHAL: Theory of functions of a real variable, three hours; Seminar, two hours.

THE Prince Jablonowski Society announces the following prize problem for 1916:

To extend the theory of linear functional differential equations in any direction. It is particularly desired to have an exhaustive treatment of some new special cases.

ON the occasion of the Australian meeting of the British Association, the University of Adelaide conferred the degree of doctor of science on Sir OLIVER LODGE, Professor T. W. EDGEWORTH, of Oxford, and Professor E. W. BROWN, of Yale.

DR. W. VOGT, of the technical school at Carlsruhe, has been appointed professor of mathematics at the University of Heidelberg.

PROFESSOR R. ROTHE, of the technical school at Hanover, has been called to the technical school at Charlottenburg, to succeed the late Professor HETTNER.

At the University of Nebraska, Professor H. T. JOHNSON has resigned to enter business. Mr. O. GISH has been appointed instructor in mathematics.

PROFESSOR WILLIAM MARSHALL, on leave from Purdue University, has returned from Europe and has been appointed assistant professor of mathematics in the University of Arizona for the year 1914–1915.

At the University of Pennsylvania, Dr. O. E. GLENN has been promoted to a full professorship of mathematics, and Dr. H. H. MITCHELL to an assistant professorship of mathematics. Dr. L. J. REED has been appointed instructor in mathematics.

At Wellesley College, Dr. CLARA E. SMITH has been promoted to an associate professorship of mathematics.

DR. H. T. BURGESS, of the University of Wisconsin, has been promoted to an assistant professorship of mathematics.

At the University of Nebraska, Dr. HENRY BLUMBERG has been promoted to an assistant professorship of mathematics.

PROFESSORS P. BOUTROUX and J. H. M. WEDDERBURN, of Princeton University, have returned to Europe and offered their services to their respective governments.

DR. C. SHAW has been appointed professor of mathematics in the University of Idaho.

DR. C. F. CRAIG, of Cornell University, who was granted leave of absence during the present academic year, has resumed his regular duties at the University.

MR. L. R. FORD, of Harvard University, has resigned, on account of the war, the Sheldon fellowship on which he was to have studied abroad, and has accepted a lectureship in mathematics at the University of Edinburgh.

MR. P. E. HENKE and Mr. L. E. WILLIAMS have been appointed instructors in mathematics at the Georgia School of Technology.

MR. W. A. REINERT has been appointed instructor in mathematics at the Michigan Agricultural College.

MR. H. D. FRARY has been appointed instructor in mathematics at the State University of Iowa.

MR. E. B. ESCOTT, of the University of Michigan, has resigned his position to enter on actuarial work.

BOOK catalogues: Galloway and Porter, Cambridge, England, catalogue 74, 412 titles.—Alexander Brunton, 54 Hanover Street, Edinburgh, catalogue of antiquarian and modern mathematical books.

NEW PUBLICATIONS.

I. HIGHER MATHEMATICS.

ÅLANDER (M.). Sur le déplacement des zéros des fonctions entières par leur dérivation. Upsala, 1914. 8vo. 63 pp. M. 2.50

BACHELIER (L.). Le jeu, la chance et le hasard. (Bibliothèque de Philosophie scientifique.) Paris, Flammarion, 1914. 8vo. 320 pp. Fr. 3.50

BAYLISS (R. W.). A first school calculus. London, Arnold, 1914. Cr. 8vo. 4s. 6d.

BERNSTEIN (B. A.). Complete set of postulates for the logic of classes expressed in terms of the operation "exception", and a proof of the independence of a set of postulates due to Del Ré. Berkeley (*Univ. Calif. Publ.*), 1914. Royal 8vo. 10 pp.

BIOCHE (CH.). Histoire des mathématiques. Paris, E. Belin, 1914. 16mo. Fr. 1.75

BJÖRNBO (A.). See WERNER.

BOMPIANI (E.). Forma geometrica delle condizioni per la deformabilità delle ipersuperficie. Roma, tip. r. accademia dei Lincei (Estr. *Rendiconti d. r. accademia dei Lincei; scienze fisiche*), 1914. 8vo. 6 pp.

BRIDGES (J. H.). The life and work of Roger Bacon: an introduction to the Opus Majus. Edited with additional notes and tables by H. G. Jones. London, Williams and Norgate, 1914. 3s

CARMICHAEL (R. D.). The theory of numbers. (Mathematical Monographs, No. 13.) New York, Wiley, 1914. 8vo. 94 pp. Cloth. $1.00

CASTELNUOVO (G.). Lezioni di geometria analytica. 3a edizione. Milano, 1914. 8vo. L. 15.00

CHERUBINO (S.). Sulle curve iperellittiche con trasformazioni birazionali singolari in sè e sui loro moduli algebrici. Torino (*Atti della r. accademia delle scienze*), tip. V. Bona, 1914. 8vo. 22 pp.

COMBEROUSSE (C. DE). Cours de mathématiques. Tome IV: Algèbre supérieure. 2e partie: Etudes imaginaires. Théorie générale des équations. 4e édition. Paris, Gauthier-Villars, 1914. 8vo. 24+831 pp. Fr. 15.00

DE MORGAN (A.). Essays on the life and work of Newton. Edited, with notes and appendices, by P. E. B. Jourdain. Chicago, Open Court, 1914. 8vo. 212 pp. $1.25

EHRIG (G.). See FRIEDRICH (M.).

ENCYCLOPÉDIE des sciences mathématiques pures et appliquées. Tome III, volume 4, fascicule 1: Quadriques, exposé d'après l'article allemand de O. Staude par A. Grévy. Paris, Gauthier-Villars, 1914. 8vo. 164 pp. Fr. 8.00

ENCYKLOPÄDIE der mathematischen Wissenschaften. Band III 1, Heft 5, pp. 761–962: Elementare Geometrie vom Standpunkte der neueren Analysis aus, von J. Sommer. Elementargeometrie und elementare nicht-Euklidische Geometrie in synthetischer Darstellung, von M. Zacharias. 1ter Teil. Leipzig, Teubner, 1914. M. 6.00

FORSYTH (A. R.). A treatise on differential equations. 4th edition. London, Macmillan, 1914. 8vo. 584 pp. 14s.

FRIEDRICH (M.). Grundzüge der analytischen Geometrie. 3te Auflage von G. Ehrig. Leipzig, J. J. Weber, 1914. 207 pp. M. 2.50

GABRILOVITSCH (L.). Ueber mathematisches Denken und den Begriff der aktuellen Form. Berlin, 1914. M. 2.50

GARBE (E.). Zur Theorie der Integralgleichungen. (Diss.) Tübingen, 1914.

GIUDICE (F.). Teoria delle sezioni coniche e nozioni di geometria descrittiva. Pavia, 1914. 8vo. 218 pp. L. 3.50

GOURSAT (E.). Lehrbuch der Analysis. Nach der 2te französischen Auflage übersetzt von F. J. Schwartz. 3 Bände. Band I. Leipzig, 1914. 8vo. 12+591 pp. M. 12.00

GRÉVY (A.). See ENCYCLOPÉDIE.

GROMMER (J.). Ganze transzendente Funktionen mit lauter reelen Nullstellen. (Diss.) Göttingen, 1914.

JONES (H. G.). See BRIDGES (J. H.).

JOURDAIN (P. E. B.). See DE MORGAN (A.).

JUEL (C.). Einleitung in die Theorie der Elementarkurven dritter und vierter Ordnung. Kopenhagen, Høst, 1914.

KROHS (G.). Dühring und Fermat. Eine gemeinverständliche und mathematische Studie zum Wolfskehlpreise nebst einer Lösung des Fermat'schen Problems. Berlin, 1914. 4to. 27 pp. M. 1.00

LAUDIEN (H.). Entwicklung willkürlicher Funktionen nach Funktionen spezieller orthogonaler und biorthogonaler Systeme. (Diss.) Breslau, 1914.

LEHMER (D. N.). List of prime numbers from 1 to 10,006,721. (Carnegie Institution, Publication no. 165.) Washington, 1914. Oblong folio. 8+133 pp.

LÉVY (F.). Sur la détermination, par les fonctions elliptiques, du nombre des classes de formes quadratiques binaires de déterminant négatif donné. (Thèse, Zurich.) Genève, A. Kündig, 1914. 8vo. 48 pp.

LOEWENBERG (G.). Was muss man von der Differential- und Integralrechnung wissen? 2te Auflage. Berlin, 1914. 8vo.

MANSION (P.). Abriss der Theorie der Hyperbelfunktionen nebst einer rein analytischen Theorie der Kreisfunktionen. Nach der dritten französischen Auflage übersetzt. Leipzig, Teubner, 1913. 8vo. 44 pp. M. 1.25

PATERNO (F. P.). Nuovi metodi d'analisi geometrografica. Part I. (*Atti del Collegio degli Ingegneri ed Architetti in Palermo.*) Palermo, Virzé, 1913. 8vo. 29 pp.

PILIZOTTI (K.). Geometrographische Untersuchungen einiger photogrammetrischer Probleme. Wien, 1912. 10 pp. M. 1.00

ROYAL SOCIETY. Catalogue of scientific papers, 1883–1900. Volume XIII, A-B. Cambridge, University Press, 1914. 98+951 pp. £2 10s.

RUSSELL (B.). Our knowledge of the external world as a field for scientific method in philosophy. Chicago and London, Open Court, 1914. 8vo. 7+245 pp. 7s. 6d.

SALOMON (C.). Questions inédites de magie arithmétique polygonale. Etoiles magiques à 8, 16 et 20 branches (24, 64 et 100 points) et rosaces hypermagiques (16, 25 et 36 points). Paris, Gauthier-Villars, 1913. 8vo. 22 pp. Fr. 1.50

SCHWARTZ (F. J.). See GOURSAT (E.).

SCHULZ (O.). Ueber Tetraeder mit rationalen Masszahlen der Kantenlängen und des Volumens. Halle, 1914. 8vo. 12+292 pp. M. 4.50

SEIDEL (R.). Ueber starre räumliche Bewegungen, deren Achsenflächen Zylinder sind. (Diss.) Dresden, 1914.

SMOLAR (G.). Beweis des grossen Fermatschen Satzes über die Gleichung $a^m + b^m = c^m$. Brandeis a. E., 1914. 8vo. 25 pp. M. 2.00

SOMMER (J.). See ENCYKLOPÄDIE.

STAUDE (O.). See ENCYCLOPÉDIE.

TERRACINI (A.). Alcune questioni sugli spazi tangenti e osculatori ad una varietà: nota I. Torino (*Atti della r. accademia delle scienze*), tip. V. Bona, 1914. 8vo. 36 pp.

THOMPSON (A. W. H.). A new analysis of plane geometry, finite and differential. Cambridge, University Press, 1914. 16+120 pp. 7s.

TSCHUPPIK (W.). Ueber den Rotationskörper $X^2Z^2 + Y^2Z^2 = K$ als Integrator. Berlin, 1914. M. 0.70

UCCELI (A.). I fondamenti della geometria di posizione. (Biblioteca del popolo no. 566.) Milano, casa ed. Sonzogno (Matarelli), 1914. 16mo. 56 pp. L. 0.20

VOSS (A.). Ueber die mathematische Erkenntnis. (Die Kultur der Gegenwart. Des Gesamtwerkes Teil III Abteilung I: Die mathematischen Wissenschaften, unter Leitung von F. Klein. 3te Lieferung.) Leipzig, Teubner, 1914. M. 5.00

WERNER OF VERNER. Ioannis Verneri de triangulis sphaericis libri quatuor. De meteoroscopiis libri sex. Cum prooemio Georgii Ioachimi Rhetici. II: De meteoroscopiis, herausgegeben von Joseph Würschmidt unter Benutzung der Vorarbeiten von Dr. A. Björnbo. Leipzig, Teubner, 1913. 8vo. 2+260 pp. M. 12.00

WILENSKY (M.). Ueber Besselsche Funktionen. (Diss., Bern.) Zurich, Leemann, 1914. 8vo. 65 pp.

WÜRSCHMIDT (J.). See WERNER.

ZACHARIAS (M.). See ENCYKLOPÄDIE.

II. ELEMENTARY MATHEMATICS.

AMATURO (M.). Alcuni teoremi sui triangoli. Napoli, tip. r. accademia delle scienze fisiche e matematiche (Estr. *Archivo del ministero della P. I.*, 1885), 1913. 8vo. 19 pp. con tavola.

BARRIOL (A.). See TRIGNART (A.).

BRAGINTON (W.). Arithmetic papers for junior pupils. With or without answers. London, Macdonald and Evans, 1914. 9d.

CARONNET (T.). Cours de trigonométrie à l'usage des candidats au Baccalauréat, à l'Ecole spéciale militaire de Saint-Cyr et à l'Institut agronomique. Paris, Gauthier-Villars, 1912. 8vo. 218 pp. Fr. 4.50

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